The Pythagorean Theorem

The Pythagorean Theorem: The side opposite the right angle in a right triangle is called the **hypotenuse**. This side is always the longest side of a right triangle. The other two sides are called the **legs** of the triangle. To find the length of any side of a right triangle, given the lengths of the other two sides, you can use the **Pythagorean Theorem**.

<table>
<thead>
<tr>
<th>Pythagorean Theorem</th>
<th>If ( a ) and ( b ) are the measures of the legs of a right triangle and ( c ) is the measure of the hypotenuse, then ( c^2 = a^2 + b^2 ).</th>
</tr>
</thead>
</table>

Find the missing length.

\[
c^2 = a^2 + b^2
\]

\[
c^2 = 5^2 + 12^2
\]

\[
c = \sqrt{169}
\]

\[
c = 13
\]

The length of the hypotenuse is 13.

**Exercises**

Find the length of each missing side. If necessary, round to the nearest hundredth.

1. \[
\begin{align*}
30 & \quad 40 \\
50 &
\end{align*}
\]

2. \[
\begin{align*}
100 & \quad 110 \\
45.83 &
\end{align*}
\]

3. \[
\begin{align*}
25 & \quad 25 \\
35.36 &
\end{align*}
\]

4. \[
\begin{align*}
14 & \quad 8 \\
16.12 &
\end{align*}
\]

5. \[
\begin{align*}
4 & \quad \sqrt{13} \\
5.57 &
\end{align*}
\]

6. \[
\begin{align*}
5 & \quad b \\
8 &
\end{align*}
\]
The Pythagorean Theorem

Right Triangles If \(a\) and \(b\) are the measures of the shorter sides of a triangle, \(c\) is the measure of the longest side, and \(c^2 = a^2 + b^2\), then the triangle is a right triangle.

Determine whether each set of measures can be sides of a right triangle.

a. 10, 12, 14
Since the greatest measure is 14, let \(c = 14\), \(a = 10\), and \(b = 12\).
\[c^2 = a^2 + b^2\] Pythagorean Theorem
\[14^2 = 10^2 + 12^2\] \(a = 10\), \(b = 12\), \(c = 14\)
\[196 = 100 + 144\] Multiply.
\[196 \neq 244\] Add.
Since \(c^2 \neq a^2 + b^2\), segments with these measures cannot form a right triangle.

b. 7, 24, 25
Since the greatest measure is 25, let \(c = 25\), \(a = 7\), and \(b = 24\).
\[c^2 = a^2 + b^2\] Pythagorean Theorem
\[25^2 = 7^2 + 24^2\] \(a = 7\), \(b = 24\), \(c = 25\)
\[625 = 49 + 576\] Multiply.
\[625 = 625\] Add.
Since \(c^2 = a^2 + b^2\), segments with these measures can form a right triangle.

Exercises
Determine whether each set of measures can be sides of a right triangle. Then determine whether they form a Pythagorean triple.

1. 14, 48, 50 \text{ yes; yes} \\
2. 6, 8, 10 \text{ yes; yes} \\
3. 8, 8, 10 \text{ no; no}

4. 90, 120, 150 \text{ yes; yes} \\
5. 15, 20, 25 \text{ yes; yes} \\
6. 4, 8, 4\sqrt{5} \text{ yes; no}

7. 2, 2, \sqrt{8} \text{ yes; no} \\
8. 4, 4, \sqrt{20} \text{ no; no} \\
9. 25, 30, 35 \text{ no; no}

10. 24, 36, 48 \text{ no; no} \\
11. 18, 80, 82 \text{ yes; yes} \\
12. 150, 200, 250 \text{ yes; yes}

13. 100, 200, 300 \text{ no; no} \\
14. 500, 1200, 1300 \text{ yes; yes} \\
15. 700, 1000, 1300 \text{ no; no}